# Two-index Mathematical Model the Distribution of the channels in Multichannel Mesh Networks 802.11 

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#### Abstract

The article introduces a two-index model of the distribution channels in the multiradio multichannel wireless mesh networking (MR-MC WMN) standard IEEE 802.11. The model describes the process of distribution channels in both homogeneous and in heterogeneous MR-MC WMN. Keywords - Wireless Mesh Network, Mathematical Model, the distribution channels, transmission range, multiradio, multichannel.


## I. Introduction

One of the most promising areas of modern telecommunications technology wireless networks are aimed at expanding the range of services provided by the user, which in turn requires an increase in productivity and improve the basic quality of service. Committing these requirements is largely due to the use of MR-MC WMN, and its performance is largely determined by the method of allocation of channels between radio network interface cards (NICs) mesh stations [1].
Thus the problem is in the selection or development of models and methods of distribution channels between the NIC stations in MR-MC WMN standard IEEE 802.11.

## II. Model of Disribution Channel

The model presented in $[2,3]$ is the most completely matches the above mentioned requirements. However, its three-index nature specifies the high dimensionality of the problem on the distribution of the channels in the mesh network solution which is necessary to provide in real time.

In accordance with the requirements there was formulated systemic nature proposed two-index mathematical model of distribution of the channels in MC-MR WMN, at the development of which the following inputs were used: $\left\{\mathrm{R}_{\mathrm{n}}, \mathrm{n}=\overline{1, \mathrm{~N}}\right\}$ - multitude of mesh stations, where N - their total quantity in mesh network; K - the total number of nonoverlapping channels used in WMN (in technology IEEE $802.11 \mathrm{~b} / \mathrm{g}$ is available $3 \div 4$ non-overlapping channels, and technology IEEE 802.11a - 12 non-overlapping channels); $\left\{\mathrm{G}_{\mathrm{z}}, \mathrm{z}=\overline{1, Z}\right\}$ multitude transmission range (TR), where $Z$ - the total number of TR to WMN, $\left|G_{z}\right|$ - capacity of $z$-th submultitude, i.e. the number of mesh stations

[^0]belonging to the z -th $\mathrm{TR} ; \mathrm{m}_{\mathrm{n}}^{*}$ - integer parameter, which characterizes the minimum of non-overlapping channels allocated to the $n$-th mesh station required number. In most cases, as a rule this parameter is equal to unity; $m_{n}$ - number supported by the NIC on n-th mesh station, which is usually equal to $1 \div 3$.

In the mathematical model the concept of matrix transmission range or TR-matrix was used, which was introduced in [2, 3], and has the view of

$$
\mathrm{D}=\left\|\mathrm{d}_{\mathrm{z}, \mathrm{n}}\right\|,(\mathrm{z}=\overline{1, \mathrm{Z}} ; \mathrm{n}=\overline{1, \mathrm{~N}})
$$

where $d_{z, n}= \begin{cases}1, & \text { if } n-t h \text { station belongs to } z-t h ~ T R ; ~ \\ 0, & \text { otherwise } .\end{cases}$
In the proposed model, similar to the model proposed in $[2,3]$, in the course of solving the problem of the distribution of the channels between mesh stations WMN we must provide the calculation of the Boolean control variable:

$$
\begin{equation*}
\mathrm{x}_{\mathrm{n}, \mathrm{k}} \in\{0,1\}(\mathrm{n}=\overline{1, \mathrm{~N}} ; \mathrm{k}=\overline{1, \mathrm{~K}}) \tag{1}
\end{equation*}
$$

where $\mathrm{x}_{\mathrm{n}, \mathrm{k}}= \begin{cases}0, & \text { if } \mathrm{n}-\text { th station does not work } \\ \text { on } \mathrm{k}-\text { th channel } ; \\ 1, & \text { if } \mathrm{k}-\text { th channel on } \mathrm{n}-\mathrm{th} \text { mesh station } \\ \text { attached onlyat one NIC. }\end{cases}$
The novelty of this model lies in the fact that, in comparison with the model proposed in [2,3], there was reduced the total number of control variables in (1) determining the order of distribution the channels (the account of the number of NIC is not used for mesh network stations).

The result of the calculation of the control variables (1) must be a partition of mesh network as a whole and each TR separately, connected to each other collision domains, within which stations operate at the same channel. Therefore, when calculating the unknown variables $\mathrm{x}_{\mathrm{n}, \mathrm{k}}$ in every single TR we must fulfill a number of important conditions of constraints:

1) Condition of the $n$-th mesh station network inclusion

$$
\begin{equation*}
\sum_{\mathrm{k}=1}^{\mathrm{K}} \mathrm{x}_{\mathrm{n}, \mathrm{k}} \geq \mathrm{m}_{\mathrm{n}}^{*} \quad(\mathrm{n}=\overline{1, \mathrm{~N}}) \tag{2}
\end{equation*}
$$

where $1 \leq \mathrm{m}_{\mathrm{n}}^{*} \leq \mathrm{m}_{\mathrm{n}}, \quad \sum_{\mathrm{k}=1}^{\mathrm{K}} \mathrm{x}_{\mathrm{n}, \mathrm{k}}$ - number of the channels, distributed for the work of one mesh station.
2) The condition of release n-th mesh station of the number channels, not exceeding number of NIC:

$$
\begin{equation*}
\sum_{\mathrm{k}=1}^{\mathrm{K}} \mathrm{x}_{\mathrm{n}, \mathrm{k}} \leq \mathrm{m}_{\mathrm{n}}(\mathrm{n}=\overline{1, \mathrm{~N}}) \tag{3}
\end{equation*}
$$

3) The condition of two mesh stations work with each other (within a TR) not more than one channel:

$$
\begin{equation*}
\sum_{\mathrm{k}=1}^{\mathrm{K}}\left[\mathrm{x}_{\mathrm{n}, \mathrm{k}} \mathrm{x}_{\mathrm{s}, \mathrm{k}}\right] \leq 1 \tag{4}
\end{equation*}
$$

for $(\mathrm{n}, \mathrm{s})$-a pair of stations, with the condition that $\mathrm{n}, \mathrm{s}=\overline{1, \mathrm{~N}}$, $\mathrm{n}, \mathrm{s} \in \mathrm{G}_{\mathrm{z}}, \mathrm{z}=\overline{1, \mathrm{Z}}$, which is introduced to eliminate the undesirable structural redundancy.
4) The condition that an arbitrary mesh station used on its channel works with at least one mesh station of its TR:

$$
\begin{equation*}
\mathrm{x}_{\mathrm{n}, \mathrm{k}} \leq \sum_{\mathrm{s}=1}^{\mathrm{N}} \mathrm{x}_{\mathrm{s}, \mathrm{k}} \quad\left(\mathrm{n}, \mathrm{~s} \in \mathrm{G}_{\mathrm{z}} ; \mathrm{z}=\overline{1, \mathrm{Z}} ; \mathrm{k}=\overline{1, \mathrm{~K}} ; \mathrm{s} \neq \mathrm{n}\right) \tag{5}
\end{equation*}
$$

where $\sum_{\mathrm{s}=1}^{\mathrm{N}} \mathrm{x}_{\mathrm{s}, \mathrm{k}}$ - the number of mesh stations in the TR $\mathrm{G}_{\mathrm{z}}$ (excluding the n -th mesh station) that work at k -th channel.
5) The condition of no effect "hidden station":

$$
\begin{equation*}
\mathrm{d}_{\mathrm{z}, \mathrm{n}} \mathrm{~d}_{\mathrm{q}, \mathrm{n}} \mathrm{x}_{\mathrm{n}, \mathrm{k}} \sum_{\mathrm{s}=1}^{\mathrm{N}} \mathrm{x}_{\mathrm{s}, \mathrm{k}} \sum_{\mathrm{r}=1}^{\mathrm{N}} \mathrm{x}_{\mathrm{r}, \mathrm{k}}=0 \tag{6}
\end{equation*}
$$

given $\mathrm{n}=\overline{1, \mathrm{~N}}, \mathrm{k}=\overline{1, \mathrm{~K}}, \mathrm{z}, \mathrm{q}=\overline{1, \mathrm{Z}}, \mathrm{s} \in \mathrm{G}_{\mathrm{z}}, \mathrm{r} \in \mathrm{G}_{\mathrm{q}}, \mathrm{s} \notin \mathrm{G}_{\mathrm{q}}$, $\mathrm{r} \notin \mathrm{G}_{\mathrm{z}}, \mathrm{n} \neq \mathrm{s} \neq \mathrm{r}$.
6) The condition of mesh network connectedness (domains of collision) in each TR:

$$
\begin{equation*}
\sum_{\mathrm{k}=1}^{\mathrm{K}} \sum_{\mathrm{n}=1}^{\mathrm{N}} \mathrm{x}_{\mathrm{n}, \mathrm{k}} \geq\left|\mathrm{G}_{\mathrm{z}}\right|+\mathrm{K}-1-\mathrm{b}\left(\mathrm{z}=\overline{1, \mathrm{Z}} ; \mathrm{n} \in \mathrm{G}_{\mathrm{z}}\right) \tag{7}
\end{equation*}
$$

given $\mathrm{b}=\left\{\begin{array}{l}\mathrm{K}-\mathrm{N}, \text { if } \mathrm{K}>\mathrm{INT}\left(\left(\sum_{\mathrm{n}=1}^{\mathrm{N}} \mathrm{m}_{\mathrm{n}}\right) / 2\right) \text {; } \\ 0 \text {, otherwise. }\end{array}\right.$
Expression $\operatorname{INT}\left(\left(\sum_{\mathrm{n}=1}^{\mathrm{N}} \mathrm{m}_{\mathrm{n}}\right) / 2\right)$ in condition-constraint
specifies the maximum number of non-overlapping channels, which may be included in the NICs mesh network stations.
7) The condition that a one of set mesh stations located at the intersection of several TR and sustainable work for at least two channels, to work on these channels in different TR:

$$
\left\{\begin{array}{l}
\mathrm{d}_{\mathrm{z}, \mathrm{n}} \mathrm{~d}_{\mathrm{q}, \mathrm{n}} \mathrm{x}_{\mathrm{n}, \mathrm{k}} \mathrm{x}_{\mathrm{n}, \mathrm{~h}}\left(\sum_{\mathrm{s}=1}^{\mathrm{N}}\left(\mathrm{~d}_{\mathrm{z}, \mathrm{~s}} \mathrm{x}_{\mathrm{s}, \mathrm{k}}\right)+\sum_{\mathrm{s}=1}^{\mathrm{N}}\left(\mathrm{~d}_{\mathrm{z}, \mathrm{~s}} \mathrm{x}_{\mathrm{s}, \mathrm{~h}}\right)\right) \times \ldots  \tag{9}\\
\ldots \times\left(\sum_{\mathrm{r}=1}^{\mathrm{N}}\left(\mathrm{~d}_{\mathrm{q}, \mathrm{r}} \mathrm{x}_{\mathrm{r}, \mathrm{k}}\right)+\sum_{\mathrm{r}=1}^{\mathrm{N}}\left(\mathrm{~d}_{\mathrm{q}, \mathrm{r}} \mathrm{x}_{\mathrm{r}, \mathrm{~h}}\right)\right)>0 \\
\mathrm{~d}_{\mathrm{z}, \mathrm{~s}} \mathrm{~d}_{\mathrm{q}, \mathrm{~s}}=0 \\
\mathrm{~d}_{\mathrm{z}, \mathrm{r}} \mathrm{~d}_{\mathrm{q}, \mathrm{r}}=0
\end{array}\right.
$$

given $\mathrm{k}, \mathrm{h}=\overline{1, \mathrm{~K}}, \mathrm{k} \neq \mathrm{h}, \mathrm{z} \neq \mathrm{q}, \mathrm{n} \neq \mathrm{s} \neq \mathrm{r}$, i.e., for example, condition means that the station is not located at the intersection TR $\mathrm{G}_{\mathrm{z}}$ and $\mathrm{G}_{\mathrm{q}}$.
8) The condition of one of the many mesh stations located at the intersection of several TR, on more than one channel.

$$
\begin{equation*}
\sum_{\mathrm{k}=1}^{\mathrm{K}} \sum_{\mathrm{n}=1}^{\mathrm{N}}\left(\mathrm{~d}_{\mathrm{z}, \mathrm{n}} \mathrm{~d}_{\mathrm{q}, \mathrm{n}} \mathrm{x}_{\mathrm{n}, \mathrm{k}}\right) \geq \sum_{\mathrm{n}=1}^{\mathrm{N}}\left(\mathrm{~d}_{\mathrm{z}, \mathrm{n}} \mathrm{~d}_{\mathrm{q}, \mathrm{n}}\right)+1 \tag{8}
\end{equation*}
$$

where $\mathrm{z}, \mathrm{q}=\overline{1, \mathrm{Z}} ; \mathrm{z} \neq \mathrm{q} ; \sum_{\mathrm{k}=1}^{\mathrm{K}} \sum_{\mathrm{n}=1}^{\mathrm{N}}\left(\mathrm{d}_{\mathrm{z}, \mathrm{n}} \mathrm{d}_{\mathrm{q}, \mathrm{n}} \mathrm{x}_{\mathrm{n}, \mathrm{k}}\right)$ - the number of the employed NIC mesh stations, which are located at the intersection TR $G_{z}$ and $G_{q} ; \sum_{n=1}^{N}\left(d_{z, n} d_{q, n}\right)$ - the number of mesh stations located at the intersection $T R G_{z}$ and $G_{q}$.
9) The condition of balancing mesh stations in the processes of creation the domains collisions depending on the territorial distance and number of TR

$$
\begin{equation*}
\sum_{\mathrm{n}=1}^{\mathrm{N}}\left(\mathrm{~d}_{\mathrm{z}, \mathrm{n}} \mathrm{x}_{\mathrm{n}, \mathrm{k}}\right) \leq \alpha \tag{10}
\end{equation*}
$$

for each given $(\mathrm{z}, \mathrm{k})$-a pair $\mathrm{z}=\overline{1, \mathrm{Z}}, \mathrm{k}=\overline{1, \mathrm{~K}}$. In the left hand side of expression (10) there is given the number of mesh stations in the z -th TR working on the k -th channel, $\alpha$ - the upper dynamically controlled threshold of the mesh stations number casually selected domain of collision in the MC-MR WMN.

At the output of the model an optimization problem related to minimizing the number of mesh stations created by domain of collisions was formulated. The optimization problem is considered relatively to the activity and territorial remoteness mesh stations and has form of $\min _{\mathrm{x}, \alpha} \alpha$.

## III. Conclusion

The paper presents a mathematical model of the distribution of channels in MR MC WMN, the computational complexity which, in $\left(\sum_{n=1}^{N} m_{n}\right) / N$ times lower than computational complexity of the known solutions proposed in [2,3]. The problem itself belongs to the class of mixed-integer nonlinear programming (MINLP) for the solution of which system of MatLab in which the program was activated minlpAssign optimization package TOMLAB was used. As a result of the proposed model using there was formed a domains of collisions connected structure, which enables data exchange between any pair of stations MC-MR WMN.

## References

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