Hypergraph Representations of Topological Model Mesh-Network IEEE 802.11

Sergii Harkusha, Olena Harkusha, Marina Ievdokymenko

Abstract - **An approach to the use of hypergraphs for modeling multi-channel multi-radio mesh-networking standard IEEE 802.11, both at the stage of task channels allocation, and when analyzing the results of decisions. This, in turn, allowed for a fuller and describe in detail all the possible configurations of mesh-network as a whole and its individual elements are represented as nodes and edges of the hypergraph. Also acquires a new formalization of the problem of determining connectivity.**

Keywords - **multi-channel multi-radio mesh-networking, hypergraph, channels allocation.**

I. Introduction

One of the effective ways to improve the performance of mesh-network standard IEEE 802.11 is the use of multi-channel (MC) multi-radio (MR) mode. The productivity of MR-MC WMN IEEE 802.11 standard is largely depends on the mechanism of frequency channels (FC) allocation [1-3].

It should be noted that traditional approaches to the synthesis of structural models based on telecommunications networks mathematical apparatus of the theory of graphs. However, graph representation MR-MC WMN with her characteristic simplicity and clarity involuntarily "calls" the basic elements of the system being simulated. Thus in modeling MR-MC WMN is necessary to use more efficient, though perhaps more complex, ways of presenting the mesh-network using topological ideas. As such, approaches can be used mathematical apparatus of hypergraphs [4, 5].

II. Hypergraphs representation multichannel mesh-networks

At the stage of allocation problem FC in MR-MC WMN each individual station is assigned a vertex hypergraph. By analogy, each individual transmission range (TR) is associated with an edge of the $z_j \in Z$ hypergraph. Then the R predicate, being incidentors hypergraph H determines whether an i-th station zone of j-th stable reception. So in case i-th mesh-station participates in the formation j-th TR, the predicate $R(n_i, z_j)$ – the true, i.e. equal to one, otherwise $R(n_i, z_j)$ – false, i.e. zero. As a result, the description of the MR-MC WMN can be performed using finite hypergraph $H = (N, Z; R)$ consisting of a pair of sets of vertices $N = \{n_i / i \in I\}$ and edges $Z = \{z_j / j \in J\}$ with binary predicates $R \dot{\Leftrightarrow} R(n_i, z_j)$ defined for all $n_i \in N$ and $z_j \in Z$. Based on this, the i-th station j-th belonging stable reception area determined incidence i-th tops j-th edge in the hypergraph H [4, 5].

Within hypergraphs describe uniquely manages to formalize rules for forming the transmission range matrix (TR-matrix), introduced in [1-3], using the incidence matrix of a hypergraph H.

$$A(H) \doteq \left\| a_{z_j, ni} \right\|, \tag{1}$$

where
$$a_{z_j, n_i} = \begin{cases} 1, & \text{if } i\text{-th station and included in the } j\text{-th TR, } i\text{.e predicate } R(n_i, z_j) = 1; \\ 0, & \text{otherwise, } i\text{.e. predicate } R(n_i, z_j) = 0. \end{cases}$$

In [1-3] solution of the problem is the calculation of the allocation of the FC boolean variable x_{n_i, k_t} characterizing the binding channel $k_t \in K$ for the mesh-station $n_i \in N$, where K – the set of non-overlapping channels.

$$x_{n_i, k_t} = \begin{cases} 1, & \text{if } i\text{-th station selected } t\text{-th non-overlapping channels;} \\ 0, & \text{otherwise}. \end{cases} \tag{2}$$

As a result of solving the problem of the channels allocation made fixing t-th channel for i-th station owned by j-th TR. Thus predicate $P(n_i, k_t, z_j)$ can be calculated from the expression:

$$P(n_i, k_t, z_j) = x_{n_i, k_t} R(n_i, z_j). \tag{3}$$

It should be noted that as a result of solving the problem of the channels allocation produced formation collision domains stations one TR, using a common channel. Therefore, each individual station $n_i \in N$ is allocation to a vertex, and each collision domain $d_u \in D$ edge of the hypergraph $G(N, D; Q)$. As a

Sergiy Garkusha - Poltava University of Economics and Trade, 3, Kovalya Str., Poltava, 36014, UKRAINE, E-mail: sv.garkusha@ukr.net
Olena Garkusha - Poltava University of Economics and Trade, 3, Kovalya Str., Poltava, 36014, UKRAINE, E-mail: av.garkusha@mail.ru
Marina Ievdokymenko - Kharkiv National University of Radioelectronics, 14, Lenina street, Kharkiv, 61166, UKRAINE, E-mail: gogolevam@gmail.com

result, use of the i-th station in the formation of the u-th collision domain is defined by a predicate $Q(n_i, d_u)$. Predicate $Q(n_i, d_u)$ in turn uniquely determined by the correspondence

$$Q(n_i, d_u) \Leftrightarrow P(n_i, k_t, z_j). \qquad (4)$$

Thus, if the i-th station, which is part of j-th TR allocated t-th channel ($P(n_i, k_t, z_j)$ =1), the station participates in the formation of u-th collision domain predicate $Q(n_i, d_u)$= 1. Otherwise, if i-th mesh-station is not included in the j-th TR or she is selected t-th non-overlapping channel ($P(n_i, k_t, z_j)$ =0), then the predicate $Q(n_i, d_u)$=0.

As an example, consider the MR-MC WMN, shown in Fig. 1, consisting of the eight stations are grouped into three TR. Said mesh-network corresponds to the hypergraph $H = (N, Z; R)$ in Fig. 2, with the set of vertices $N = \{n_1, n_2, \ldots, n_8\}$, the set of edges $Z = \{z_1, z_2, z_3\}$ and a predicate R that determines membership of a particular station to any TR. For example the predicates $R(n_1, z_1)$, $R(n_2, z_1)$, $R(n_3, z_1)$, $R(n_3, z_2)$, $R(n_4, z_1)$, $R(n_4, z_2)$, $R(n_4, z_3)$, $R(n_5, z_2)$, $R(n_6, z_1)$, $R(n_6, z_3)$, $R(n_7, z_3)$, $R(n_8, z_3)$, are the true, i.e. a_{z_j, n_i} =1, and in other cases, the predicates are false, i.e. a_{z_j, n_i} = 0.

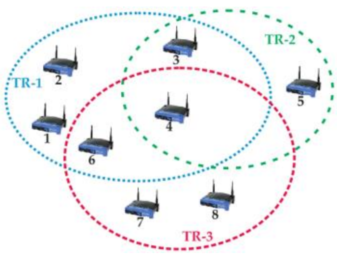

Fig 1. One possible configuration mesh-network

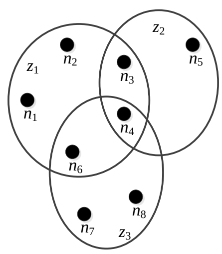

Fig. 2 Hypergraphs representation mesh-network

Mesh-network presented in Fig. 1 can be described by the following matrix of incidence (TR-matrix):

$$A(H) = \begin{Vmatrix} 1 & 1 & 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 1 \end{Vmatrix}.$$

As a result of solving the problem of the distribution of the three non-overlapping FC ($K = \{k_1, k_2, k_3\}$) using the model balances the number of stations created by collision domain [1-4], was obtained mesh-network presented in Fig. 3. Mesh-network shown in Fig. 3 corresponds hypergraph $G = (N, D; Q)$ in Fig. 4, with a set of vertices $N = \{n_1, n_2, \ldots, n_8\}$, the set of collision domains $D = \{d_1, d_2, d_3, d_4\}$ and predicate $Q(n_i, d_u)$.

As the performed in [1] analysis, reducing the number of stations included in each TR results in better performance mesh-network because of solving the problem of the channels allocation. Number of stations included in the reception area of sustainable mesh-network, using a mathematical apparatus of the theory of hypergraphs can be assessed by determining the set of vertices incident to each edge $z_j \in Z$ [4, 5]:

$$N(z_j) \doteq N_H(z_j) \doteq \{n_i \in N / R(n_i, z_j)\}. \qquad (5)$$

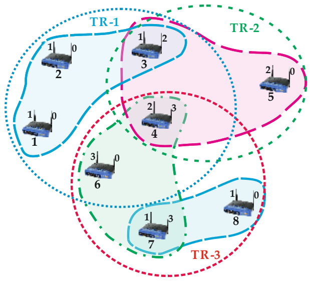

Fig. 3 Example of solving the problem of the distribution three non-overlapping FC

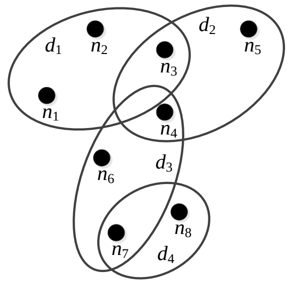

Fig. 4 Hypergraphs representation distribution problem solutions FC

In addition, each station mesh-network can simultaneously be members of multiple TR, then each vertex $n_i \in N$ of a hypergraph H can be attributed set of all incident edges represented as

$$Z(n_i) \doteq Z_H(n_i) \doteq \{ z_j \in Z / R(n_i, z_j)\}. \qquad (6)$$

As an example, consider a mesh-network shown in Fig. 1, as well as its hypergraphs representation (Fig. 2). Stations in the example mesh-network correspond to the following degrees of the vertices of the hypergraph: $|Z(n_1)| = 1$, $|Z(n_2)| = 1$, $|Z(n_3)| = 2$, $|Z(n_4)| = 3$, $|Z(n_5)| = 1$, $|Z(n_6)| = 2$, $|Z(n_7)| = 1$, $|Z(n_8)| = 1$. As can be seen from the above example, the definition of the degree of vertices of the hypergraph to determine the location of a particular station in the entire configuration mesh-network. So station №1, №2, №5, №7 and №8 have value equal to one degree of a vertex, the station №3 and №6 – two and station №4 – three. Degree of a vertex determines the number of TR, which include the station, and also determines the importance of the station, while ensuring connectivity mesh-network.

As for determining the degrees of edges, for this example, they take the following values: $|N(z_1)| = 5$, $|N(z_2)| = 3$ and $|N(z_3)| = 4$.

The physical meaning of the degree edges for mesh-network is that it shows the number of stations forming a particular zone of stable reception. For example, a TR-1 is formed by five mesh- stations, TR-2 – the three mesh-stations, and TR-3 – four mesh stations. In this pair of mesh-stations belonging to the same TR, by analogy with the vertices of the hypergraph, united by an edge are called adjacent.

In order to determine the inhomogeneity TR mesh-network, described using the theory of hypergraphs can be used concept h-uniformity. So if in hypergraph H degree of any j-th edge is equal to h ($\left|N\left(z_j\right)\right| = h$), the hypergraph H called homogeneous (h-uniformly) [4].

It follows that if the mesh-network can be provided in the form of h-homogeneous hypergraph, such as mesh-network is h-homogeneous, which parameter h indicates the number of stations included in each TR. When evaluating network mesh-connected sets $N \bigcup Z$ elements of the corresponding hypergraphs $H = \left(N, Z; R\right)$ can be divided into parts, called components. The number of components will be denoted as $\chi\left(H\right)$. In the case when the hypergraph is present only one component, for example, $\chi\left(H\right)=1$, is called connected hypergraph [4, 5]. Otherwise, hypergraph is disconnected. Therefore, if mesh-network is presented in the form of a connected hypergraph, it is also connected.

In order to determine connectivity, consider mesh-network configuration shown in Fig. 1. Mesh network shown in Fig. 1 is 2-connected, since the emergence of several components is the result of removing the station №4, as well as any of the stations №3 or №6. Removal of the stations №4 and №3 formed two components, the first of which consists of a number of №1, №2, №6, №7, №8, and the second part of the station №5. If you delete stations №4 and №6 formed two components, one of which consists of №1, №2, №3, №5, and the second of stations №7 and №8.

Deeper analysis of mesh-connected network can be achieved by determining the degree of overlap of paired (connected) TRs. Since the composition of the two individual TRs at $J > 2$, included only a portion of mesh-station network, we use the notion subhypergraph. In this subhypergraph generated by the set of vertices N', called hypergraph $H' = \left(N', Z'; R'\right)$, where $Z' = \left\{ z_j' : z_j' = z_j \bigcap N' \neq 0,\ z_j \in Z \right\}$. Since the degree of overlap is determined for the two TRs subhypergraph can then be represented as $H_{c,v} = \left(N', Z_{c,v}; R'\right)$, where $c, v \in Z$, $c \neq v$. By analogy with the definition of the entire mesh-connected network, any two TRs $b+1$-connected if they retain this property by removing b stations.

As an example, consider the possible configuration of mesh-network shown in Fig. 1. Subhypergraph $H_{1,2} = \left(N', Z_{1,2}; R'\right)$ where $N' = \left\{n_1, n_2, n_3, n_4, n_5, n_6\right\}$, $Z_{1,2} = \left\{z_1, z_2\right\}$, formed stable TR-1 and TR-2 is 2-connected, since the formation of several (two) component occurs in the case of removal in mesh-network station №3 and №4. By analogy, we define the degree of connectedness of the other pairs of TR. So subhypergraph $H_{1,3} = \left(N', Z_{1,3}; R'\right)$ is 2-connected and subhypergraph $H_{2,3} = \left(N', Z_{2,3}; R'\right)$ - 1-connected.

III. Conclusion

An approach to the use of hypergraphs for modeling MR MC mesh-networking standard IEEE 802.11. This, in turn, allowed for a fuller and describe in detail all the possible configurations of mesh-network as a whole and its individual elements are represented as nodes and edges of the hypergraph. Also acquires a new formalization of the problem of determining connectivity. Compared to using graph representation of a possible configuration mesh-network, no need to search for independent paths between all pairs of vertices. When using a solution approach hypergraphs connectivity problem reduces to finding the maximum number of stations whose removal would lead to the division of mesh-network into several disconnected components. Using hypergraphs also determine a location of the station with the mesh-network, unlike a graph representation, which spontaneously "equalizes" the main elements of the system.

References

[1] Garkusha S. Analysis the Results of Frequency Planning in Mesh Networking Standard IEEE 802.11 // *Modern Problems of Radio Engineering, Telecommunications and Computer Science. Proceedings of the international Conference TCSET'2012.* – Lviv-Slavske, Ukraine, 21-24 February 2012: Publishing House of Lviv Polytechnic, 2012. – P. 285-286.

[2] Lemeshko A., Garkusha S., Abed A.H. Two-index Mathematical Model of Channels Distribution in Multichannel Mesh Networks 802.11 // *Modern Problems of Radio Engineering, Telecommunications and Computer Science. Proceedings of the international Conference TCSET'2012.* – Lviv-Slavske, Ukraine, 21-24 February 2012: Publishing House of Lviv Polytechnic, 2012. – P. 279-280.

[3] Garkusha S., Yevdokimenko M.A. Classification and Analysis of Methods of the Distribution Channels in Multichannel Mesh Networks IEEE 802.11// *Modern Problems of Radio Engineering, Telecommunications and Computer Science. Proceedings of the international Conference TCSET'2012.* – Lviv-Slavske, Ukraine, 21-24 February 2012: Publishing House of Lviv Polytechnic, 2012. – P. 273-274.

[4] Berge C. Graphs and Hypergraphs. – *New York: Elsevier*, 1973. – 528 p.

[5] Berge C. Hypergraphs: The Theory of Finite Sets. – *Amsterdam, Netherlands: North-Holland*, 1989. – 256 p.