Electron combination scattering of light in conductors with magnetoimpurity states

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(Submitted April 18, 1995)
Fiz. Nizk. Temp. **21**, 947–950 (September 1995)

The differential cross section for inelastic scattering of light by single-particle and collective excitations in conductors with magnetoimpurity electron states is calculated. New series of lines are observed in the energy spectrum of scattered radiation. The cross sections are calculated numerically for semimetals and degenerate semiconductors. © *1995 American Institute of Physics.*

Raman scattering is one of basic methods for investigating electron excitations in solids.[1–5] This method was used by one of the authors[6] in an analysis of properties of magnetoplasma waves in metals and degenerate semiconductors with magnetoimpurity electron states.[7] It is shown that resonant transitions to Landau levels of electrons localized in the field of impurity atoms lead to strong damping of magnetoplasma waves. The damping decrement as a function of the wave frequency ω has asymmetric peaks at transition frequencies ω_s. The peaks are displaced towards the region of high frequencies $\omega > \omega_s$. The spectral regions with $\omega < \omega_s$ below resonant frequencies have not been studied. However, these regions might contain new spectral branches of longitudinal magnetoplasma waves[8] similar to transverse magnetoimpurity waves.[9] In this paper, we consider inelastic light scattering by these waves as well as by single-particle excitations localized at magnetoimpurity levels of electrons. The results obtained by us are applicable not only to magnetoimpurity states, but also to intrinsic quasi-local electron states[10] existing in zero magnetic field also. The model and the calculation technique are described in Refs. 7,9. The long-range Coulomb interaction of electrons is taken into account in the random-phase approximation.[2]

In the random-phase approximation, the differential cross section for light scattering into the solid angle dO in the frequency interval $d\omega$ associated with electron density fluctuations is given by[2]

$$\frac{d^2\sigma}{dO\,d\omega} = -\frac{r_0^2 q^2}{4\pi^2 e^2}\frac{\omega_2}{\omega_1} \times (\mathbf{l}_1\cdot\mathbf{l}_2)^2[1-\exp(-\omega/T)]^{-1}\,\mathrm{Im}\frac{q^2}{\sum_{ik} q_i\varepsilon_{ik}q_k}, \quad (1)$$

where $r_0 = e^2/mc^2$ is the classical radius of the electron with effective mass m, $q = k_1 - k_2$ and $\omega = \omega_1 - \omega_2$ are the changes in the momentum and energy of radiation due to scattering, $\mathbf{l}_1$ and $\mathbf{l}_2$ are polarization vectors of incident and scattered photons, $\varepsilon_{ik}(q,\omega)$ is the permittivity tensor for electrons in an impurity sample in a magnetic field, and T the temperature. The quantum-mechanical constant and the sample volume are assumed to be equal to unity. We assume that the scattering vector $\mathbf{q}$ is perpendicular to the magnetic field $\mathbf{H}\|\mathbf{z}$. Such a geometry is convenient since collisionless damping of magnetoplasma waves propagating at right angles to the magnetic field is not observed.[2,11] In this case, formula (1) contains only the longitudinal (relative to $\mathbf{q}$) component $\varepsilon(q,\omega)$ of the permittivity tensor.

Magetoimpurity states lead to the emergence of a resonant contribution to the longitudinal component of the permittivity tensor. In the vicinity of frequencies of resonant electron transitions from Landau levels to magnetoimpurity levels, this contribution is given by

$$\delta\varepsilon_k = \left(\frac{\omega_p}{\omega_k}\right)^2 a_k \left(\frac{\omega_k}{\omega - \omega_k + i\Gamma}\right)^{1/2}, \quad (2)$$

where ω_p is the plasma frequency, $\omega_k = k\Omega - \omega_0$ are resonance frequencies, Ω is the cyclotron frequency, ω_0 the separation between magnetoimpurity levels of width Γ and the nearest above-lying Landau level, $k = 1,2$ the number of a resonance, and

$$a_k = \left(\frac{m}{2}\right)^{3/2} \frac{n_i}{n_e}\frac{\Omega^2}{\pi\omega_k^{1/2}} \sum_N r_N [f(\varepsilon_N - \omega_k) - f(\varepsilon_N)] \times \left[\frac{N-k}{(\omega_k+\Omega)^2} + \frac{N-k+1}{(\omega_k-\Omega)^2}\right] \quad (3)$$

are oscillator forces of resonant transitions. Here n_e and n_i are the concentrations of electrons and impurity atoms,

$$\varepsilon_N = \Omega\left(N + \frac{1}{2}\right) - \omega_0$$

is the position of the Nth magnetoimpurity level, f the Fermi function, and r_N the residue of the amplitude of electron scattering by an impurity atom relative to the pole $\varepsilon_N - i\Gamma$. The quantity ω_0 is defined as

$$\omega_0 = \frac{1}{2}\Omega\left(\frac{a}{l}\right)^2,$$

where a is the scattering length and l the minimum magnetic length. The widths of magnetoimpurity levels

$$\Gamma_N = 2\omega_0\left(\frac{\omega_0}{\Omega}\right)^{1/2} \sum_n (n-n)^{-1/2}$$

and the residues r_N weakly depend on the number N, and this dependence will be disregarded. If $a \ll l$, the residue r is given by

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$$r=2^{5/2}\pi\omega_0^{3/2}/m^{3/2}\Omega.$$

In zero magnetic field, we have

$$r=2\pi\Gamma/mk_r,$$

where $k_r=(2m\varepsilon_r)^{1/2}$, ε_r being the position of a quasi-local level. The summation in formula (3) is carried out over the numbers of magnetoimpurity levels participating in resonant transitions at a given frequency ω_k. The branch of the square root in (2) is chosen so that the imaginary component of permittivity responsible for wave absorption is positive. In the long-wave limit $qr_H \ll 1$ under investigation (r_H is the Larmor radius), the dependence of a_k on q can be neglected. A contribution similar to (2) is observed near the frequencies $\omega_s=\omega_0+s\Omega(s=0,1,...)$ of resonant transitions of electrons from magnetoimpurity levels to Landau levels. This contribution is given in Ref. 8 and will not be given here.

The term (2) should be taken into account in formula (1). If we neglect the Coulomb interaction of electrons, formula (1) gives the cross section for scattering accompanied by one- particle excitations of electrons from Landau levels to magnetoimpurity levels:

$$\frac{d^2\sigma}{dOd\omega}=\frac{r_0^2q^2}{4\pi^2e^2}\frac{\omega_2}{\omega_1}(\mathbf{l}_1\cdot\mathbf{l}_2)^2(1-e^{-\omega/T})^{-1}\,\mathrm{Im}\,\varepsilon(q,\omega). \tag{4}$$

For the dimensionless cross section

$$h=\frac{d^2\sigma}{dOd\omega}\frac{8\pi e^2}{r_0^2q^2}\frac{\omega_1}{\omega_2}\left(\frac{\omega_+}{\omega_p}\right)^2(\mathbf{l}_1\cdot\mathbf{l}_2)^{-2}, \tag{5}$$

($\omega_+=\sqrt{\omega_p^2+\Omega^2}$ is the upper hybrid frequency), we obtain the following expression in the vicinity of the frequency ω_k:

$$h_k=\frac{2}{\pi}\left(\frac{\omega_+}{\omega_k}\right)^2 a_k\,\mathrm{Re}\left(\frac{\omega_k}{\omega_k-\omega-i\Gamma}\right)^{1/2}, \tag{6}$$

in which we assume that $T\ll\omega$. This formula shows that the energy spectrum for scattered radiation at frequencies ω_k contains asymmetric peaks displaced towards low frequencies. Asymmetric lines displaced towards high frequencies exist at frequencies ω_s corresponding to electron transitions from magnetoimpurity levels to Landau levels. In the vicinity of this frequency, the cross section differs from (6) in the sign of the radicand.

In order to estimate the intensity of these lines, we calculate the dependence of h on $x=\omega/\omega_1-1$ near the frequency $\omega_1=\omega_0+\Omega$ corresponding to transitions from a magnetoimpurity level to a Landau level. We use the values of the parameters

$$m=10^{-29}\mathrm{g};\quad a=10^{-6}\ \mathrm{cm};\quad n_i=10^{16}\ \mathrm{cm}^{-3},$$

typical of doped semimetals and degenerate semiconductors such as InSb, InAs, and GaAs. Electron excitations in these materials are intensely studied using the Raman scattering technique.[2,3,5,11] The results of calculations for $H=3\cdot10^4$ Oe are shown in Fig. 1. Curve *1* corresponds to $n_i/n_e=1$, $\nu=10^{11}$ s, while curve 2 corresponds to $n_i/n_e=0.5$, $\nu=10^{12}$ s, where ν is the frequency of electron collisions with impurities and other scatterers, which is associated with potential scattering. For such values of parameters, we obtain

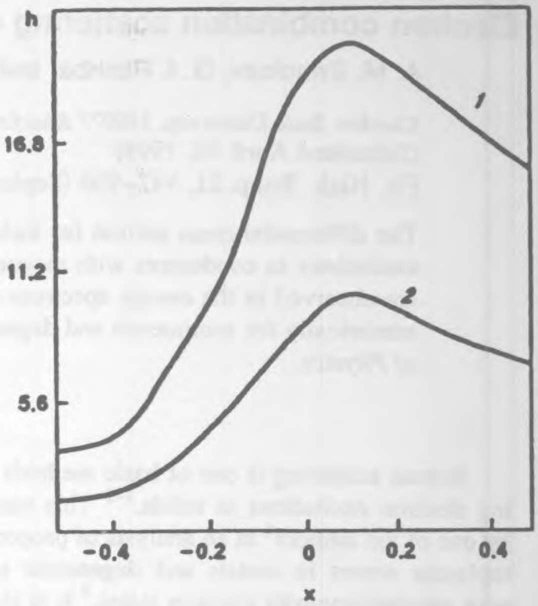

FIG. 1. Energy spectrum of one-particle scattering of light near the frequency $\omega_1=\omega_0+\Omega$ of electron transitions from a magnetoimpurity level to a Landau level.

$$\omega_p=5.4\cdot10^{13}\ \mathrm{s}^{-1};\quad \Omega=4.8\cdot10^{13}\ \mathrm{s}^{-1};$$
$$\omega_+=7.2\cdot10^{13}\ \mathrm{s}^{-1};\quad \omega_0=1.1\cdot10^{13}\ \mathrm{s}^{-1};$$
$$\omega_1=5.9\cdot10^{13}\ \mathrm{s}^{-1};\quad \Gamma/\omega_0=0.9.$$

The ratio of the maximum value of cross section in Fig. 1 to the maximum cross section for scattering by the longitudinal magnetoplasma mode[2] at the frequency ω_+ is equal to $6.5\cdot10^{-2}$ in the first case and $3.3\cdot10^{-1}$ in the second.

Let us consider the contribution of collective excitations of the electron system of the conductor to the light scattering cross section (1). This contribution is connected with zeros of the longitudinal dielectric function. The inclusion of term (2) in the dispersion equation $\varepsilon(q,\omega)=0$ leads to the emergence of new branches in the spectrum of longitudinal magnetoplasma waves propagating at right angles to the magnetic field in the spectral regions $\omega<\Omega$ (for $k=1$) and $\omega>\omega_+(k=2,3,...)$. These branches weakly attenuate in the transparency bands $[\omega_k,\omega_k(0)]$ between the resonant frequencies ω_k and the frequencies $\omega_k(0)$ of natural oscillations of electrons in the magnetic field on account of magnetoimpurity states. The energy–momentum relation for waves in the kth band has the form

$$\omega_k(q)=\omega_k\left\{1+a_k^2\left(\frac{\omega_p}{\omega_k}\right)^4\left(\frac{\omega_k^2-\Omega^2}{\omega_k^2-\omega_+^2}\right)^2\right.$$
$$\left.\times\left[1-\frac{3(qv_F\omega_p)^2}{5(\omega_k^2-\omega_+^2)(\omega_k^2-4\Omega^2)}\right]^{-2}\right\}, \tag{7}$$

where v_F is the Fermi velocity for electrons. In the long-wave limit, the dispersion of these waves is normal for $\omega_k>2\Omega$ and anomalous when $\omega_k<2\Omega$. The damping of waves is determined by electron collisions and by the width of a magnetoimpurity level. The damping decrement is given by

$$\gamma_k(\omega)=\left[\nu\frac{\omega^2+\Omega^2}{2\omega^2}+\Gamma\frac{\omega_k^3(\omega^2-\omega_+^2)^3}{4a_k^2\omega\omega_p^6(\omega^2-\Omega^2)}\right]\times\left[1+\frac{\omega_k^3(\omega^2-\omega_+^2)^3}{4a_k^2\omega\omega_p^6(\omega^2-\Omega^2)}\right]^{-1}. \quad (8)$$

The widths of transparency bands are given by

$$\delta\omega_k=\omega_k(0)-\omega_k=\omega_k a_k^2\left(\frac{\omega_p}{\omega_k}\right)^4\left(\frac{\omega_k^2-\Omega^2}{\omega_k^2-\omega_+^2}\right)^2. \quad (9)$$

For the values of parameters given above, from formulas (8) and (9) we obtain $\gamma_1=1.8\cdot 10^{11}\ s^{-1}, \delta\omega_1=7\cdot 10^{14}\ s^{-1}$ in the first case and $\gamma_1=1.3\cdot 10^{12}\ s^{-1}, \delta\omega_1=1.7\cdot 10^{14}\ s^{-1}$ in the second case. Magnetoimpurity states affect the spectrum of the longitudinal magnetoplasma mode with frequency $\omega\geqslant\omega_+$ only slightly.[2] The damping of this mode was considered in Ref. 6

The differential cross section for light scattering accompanied by the emission of a wave quantum with spectrum (7) is given by

$$\frac{d^2\sigma_k}{dOd\omega}=\frac{r_0^2q^2}{2\pi^2e^2}\frac{\omega_2}{\omega_1}(\mathbf{l}_1\cdot\mathbf{l}_2)^2a_k^2\frac{\omega_p^4}{\omega_k^3}\left(\frac{\omega_k^2-\Omega^2}{\omega_k^2-\omega_+^2}\right)\times\left[1+\frac{4a_k^2\omega_p^6(\omega_k^2-\Omega^2)}{\omega_k^2(\omega_k^2-\omega_+^2)^3}\right]^{-1}\frac{\gamma_k}{[\omega-\omega_k(q)]^2+\gamma_k^2}. \quad (10)$$

This expression shows that the widths of a series of symmetric Stokes peaks existing at the frequencies of longitudinal magnetoplasma waves with spectrum (7) are equal to damping decrements (8) of the waves. In the case of low temperatures under investigation, the intensities of anti-Stokes lines are low. Similar peaks must also exist within transparency bands for waves weakly attenuating near the frequencies of resonant electron transitions from magnetoimpurity levels to Landau levels, which are considered in Ref. 8.

Figure 2 shows the dependence of the dimensionless cross section h (5) for scattering on $x=\omega/\omega_1(q)-1$ within the band with $k=1$. Curves *1* and 2 correspond to two cases described above. The ratio of the maximum value of cross section (10) to the maximum cross section for scattering by the longitudinal magnetoplasma mode in these cases is equal to $7.9\cdot 10^{-2}$ and 0.11, respectively. Experimental observation

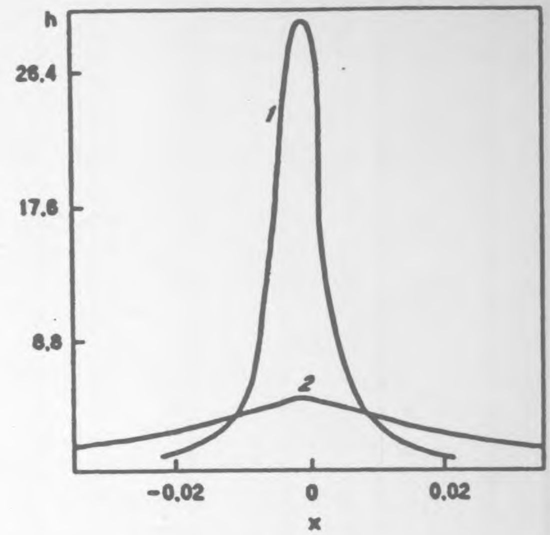

FIG. 2. Frequency dependence of the differential cross section for light scattering by longitudinal magnetoplasma waves with spectrum (7) near the frequency $\Omega-\omega_0$.

of the peaks (6) and (10) would make it possible to determine the characteristics of magnetoplasma states for electrons and new types of magnetoplasma waves.

This research was financed by the Ukrainian State Committee on Science and Technology and by the Ministry of Education of the Ukraine.

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Translated by R. S. Wadhwa