Coupled electromagnetic and acoustic waves in metals with quasilocal electron states in a magnetic field

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The coupling of transverse electromagnetic and acoustic waves in metals with quasilocal electron states is considered. It is shown that the interaction between circularly polarized waves with plus polarization (antihelicons) and sound leads to the formation of coupled waves and to their mutual transformation. The spectrum and damping factor of coupled waves are calculated.

Resonance interaction between electromagnetic and acoustic waves in metals was studied in Refs. 1–4, where it was shown that resonance and coupled waves are formed in a strong magnetic field when the characteristic radius of the electron orbit $\mathscr{R}=v_F/\Omega$ (v_F is the Fermi velocity and Ω the cyclotron frequency) is small in comparison with the length of a weakly attenuating helical electromagnetic wave and its phase velocity coincides with the velocity of sound. The observed effects are accompanied by transverse vibrations of the electron system as well as of the ion lattice. For $\mathbf{k}\|\mathbf{H}$ ($\mathbf{H}$ is the magnetic field and $\mathbf{k}$ the wave vector), the dispersion equation leads to the existence of two transverse, circularly polarized waves, a resonance wave with minus polarization and a nonresonance wave with plus polarization. Resonance takes place under the condition $\omega\Omega=(\omega_p s_t/c)^2$, where ω is the wave frequency, ω_p the plasma frequency of electrons, s_t the transverse sound velocity, and c the velocity of light in vacuum. The wave vector of the nonresonance electromagnetic wave is imaginary, hence it attenuates over the wavelength. For the corresponding acoustic wave, the velocity dispersion and attenuation are found to be insignificant.[4]

Collisions of electrons with impurity atoms were taken into account in Refs. 1–4 by introducing the phenomenological constant ν, viz., the collision frequency. At the same time, impurity atoms in a metal can scatter electrons resonantly and form quasilocal states.[5] Quasilocal states of electrons stimulated by the magnetic field at the impurities were termed magnetic impurity states.[6,7] Transparency bands, which are also called magnetic impurity bands, are formed near the frequencies of resonance transitions of electrons between magnetic impurity levels and Landau levels.[8] Waves of this type also exist in the case when electron localization is caused by impurity atoms only and is not associated with the magnetic field. Results of investigations of the properties of electromagnetic waves in metals with intrinsic quasilocal states of electrons trapped by an impurity in zero magnetic field are presented in Ref. 9. As a result of localization of electrons by impurity atoms, waves with plus polarization (antihelicons) may propagate in the vicinity of the resonance frequency ω_0 of electron transitions from a quasilocal level to a Landau level.[9] Such waves cannot propagate if there are no quasilocal states in the electron spectrum. Estimates show that the velocity of an antihelicon may be found to be nearly equal to the transverse sound velocity in the metal. This means that antihelicons can be associated with the transverse sound.

In this paper, we present the results of investigations of the characteristics of coupled electromagnetic and acoustic waves in metals with quasilocal electron states. One group of carriers with an isotropic spectrum in a weakly doped sample is considered in a strong magnetic field. It is assumed that $\mathbf{k}\|\mathbf{H}$. The results are presented in terms of the characteristics of the quasilocal state, viz., the pole $\varepsilon_r - i\Gamma$ of the amplitude of resonance scattering of electrons by an isolated impurity atom (ε_r is the resonance position and Γ its halfwidth), and the residue r of the scattering amplitude at the pole. These quantities can be either calculated by specifying the scattering potential or determined experimentally.

A consideration of the scattering of electrons by impurity atoms leads to the emergence of resonance terms in the transverse components of the dynamic conductivity tensor $\sigma_{ik}(\mathbf{k},\omega)$.[8,9] If the quasilocal level ε_r lies below the Fermi boundary ε_f (e.g., in Bi with Pb or Sn impurities),[10] the frequencies of resonance transitions of electrons from a quasilocal level to Landau levels are $\omega_s=\omega_0+s\Omega$, where ω_0 is the separation between the quasilocal level and the level closest to the Fermi boundary, and $s=0,1,...$ is the resonance number. Near this frequency ($|\omega-\omega_s|\ll\Omega$), the circular components of the rf conductivity in the longwave approximation ($kR\ll 1$) contain a contribution[9]

$$\delta\sigma_{\pm}^{(s)}=i\frac{\omega_p^2}{4\pi\omega_s}\alpha_{\pm}^{(s)}\left(\frac{\omega_s}{\omega-\omega_s+i\Gamma}\right)^{1/2}, \tag{1}$$

where

$$\alpha_{\pm}^{(s)}=2\left(\frac{m}{2}\right)^{3/2}\frac{r}{\pi}\frac{n_i}{n_e}\frac{\Omega^2}{\omega_s^{5/2}}\left[f(\varepsilon_r)-f(\varepsilon_r+\omega_s)\right]\times\begin{cases}\dfrac{N+s+1}{(1+\Omega/\omega_s)^2}\\[2ex]\dfrac{N+s}{(1-\Omega/\omega_s)^2}\end{cases} \tag{2}$$

is the oscillator energy of the resonance transition. Here m is the effective mass of an electron, n_e and n_i are the concentrations of the electrons and impurity atoms respec-

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tively, N is the number of filled Landau levels, f the Fermi function, and $\hbar=1$. The root singularity in (1) is associated with the singularity of the electron density of states at the Landau level, while the difference between the Fermi functions in (2) takes into account the Pauli exclusion principle. Formula (1) is also valid for transitions of electrons from magnetic impurity levels to Landau levels. In this case, the residue of the scattering amplitude at the pole is given by

$$r=\frac{2\pi}{\Omega}\left(\frac{2\omega_0}{m}\right)^{3/2}$$

while the oscillator energy (2) contains summation over the numbers of magnetic impurity levels participating in transitions at a frequency ω_s.[8]

The dispersion relation for coupled electromagnetic and acoustic waves has the form[4]

$$\left(1-\frac{c^2k^2}{4\pi\omega\sigma_\pm}\right)(k^2s_t^2-\omega^2)=-\frac{H^2k^2}{4\pi\rho}, \tag{3}$$

where ρ is the density of the metal. For $|\omega+i\nu|\ll\Omega$, only one resonance exists at a frequency ω_0 ($s=0$) if the quasilocal level lies just below the Fermi surface. Let us consider the contribution (1) in the dispersion Eq. (3) for waves with a positive polarization in the vicinity of this frequency. As a result, Eq. (3) takes the form

$$[\omega^2-u^2k^2(1+i\tilde{\gamma})^{-1}](k^2s_t^2-\omega^2)=-\frac{H^2}{4\pi\rho}k^2\omega^2. \tag{4}$$

Here,

$$u^2(\omega)=\left(\frac{\omega c}{\omega_p}\right)^2(\alpha_+^{(0)})^{-1}\left(1-\frac{\omega}{\omega_0}\right)^{1/2}\times\left[1-\frac{\omega}{\Omega}(\alpha_+^{(0)})^{-1}\left(1-\frac{\omega}{\omega_0}\right)^{1/2}\right]^{-1}$$

is the square of the phase velocity of an antihelicon when there is no interaction with sound, $\tilde{\gamma}=\gamma/2(\omega_0-\omega)$, γ being the attenuation of the antihelicon.[9] The right-hand side of Eq. (4) is connected with inductive interaction[4] which vanishes as $\rho\to\infty$. Consequently, the solutions of the dispersion relation (4) are an antihelicon which attenuates weakly in the transparency band $[\omega(0),\omega_0]$, and a shear acoustic wave. Here, $\omega(0)$ is the limiting frequency of an antihelicon.[9]

The solution of the dispersion relation (4) for resonance waves with positive polarization is given by

$$k_\pm^2=\left(\frac{\omega}{s_t}\right)^2\{1+\tfrac{1}{2}(\Delta+i\tilde{\gamma})\pm\tfrac{1}{2}((\Delta+i\tilde{\gamma})^2+4\delta^2)^{1/2}\}, \tag{5}$$

where $\Delta=s_t^2/u^2-1$, $\delta^2=H^2/4\pi\rho s_t^2$, and $\tilde{\gamma}$ are small quantities. The signs $\pm$ in (5) correspond to two branches of the spectrum of coupled waves, viz., the low-frequency branch and the high-frequency branch. Resonance takes place when the phase velocity of an antihelicon approaches the velocity of sound. The resonance condition has the form

$$\omega^2(1-\omega/\omega_0)^{1/2}=\alpha_+^{(0)}(\omega_p s_t/c)^2.$$

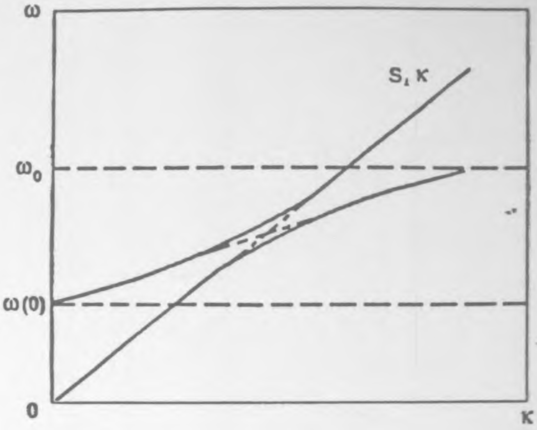

FIG. 1. Spectral branches of coupled waves in metals with quasilocal electron states.

The dispersion relation and wave attenuation in the vicinity of the resonance ($\Delta^2\ll4\delta^2$) are defined by the expression

$$k_\pm^2=(\omega/s_t)^2[1\pm\delta+i\tilde{\gamma}/2], \tag{6}$$

Away from the resonance ($\Delta^2\gg4\delta^2$), the asymptotic forms of the low-frequency branch can be presented as

$$k_+^2=(\omega/s_t)^2[1+\Delta+i\tilde{\gamma}],\quad \Delta>0, \tag{7}$$

$$k_+^2=(\omega/s_t)^2[1-\delta^2/\Delta+i\tilde{\gamma}\delta^2/\Delta^2],\quad \Delta<0, \tag{8}$$

The spectrum of the high-frequency branch is defined by formula (8) for $\Delta>0$, and coincides with formula (7) for $\Delta<0$.

Figure 1 shows the dependence of the frequency on the wave vector for a resonance wave with plus polarization. A consideration of the interaction of conduction electrons with sound near the resonance removes the degeneracy and causes a splitting of the branches. For positive values of Δ, the low-frequency wave is an antihelicon, but is transformed into an acoustic wave for negative values of Δ. Conversely, the high-frequency wave, which is an acoustic wave for $\Delta>0$, is transformed into an antihelicon for $\Delta<0$. Resonance takes place as $\Delta\to0$, and it becomes impossible to separate an acoustic wave from an antihelicon. Thus, the interaction of sound with antihelicons propagating in metals with quasilocal electron states leads to the formation of coupled waves and to their mutual transformation.

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