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# PROBLEM OF STOCHASTIC CONTROL OF ENTERPRISE

***Introduction.*** In this work we extend the approach of the previous researches to the measurement feedback case. We remove the assumption that the state of the system is available for feedback and show how algorithms from the previous researches can be used in the measurement feedback case. We derived solvability conditions for the problem but analytical computation of the optimal controller turned out to be extremely difficult task. The feasibile approach is to use model predictive control technique. So far, we have obtained several computational algorithms for model predictive control of constrained systems that are subject to stochastic disturbances. These results have been based on the assumption that all states of the plant are available for feedback.

***Resultst.*** In this scientific work, we consider the more general case in which we assume that output of the plant is measured and available for feedback. In this case, static feedbacks are no longer sufficient and we need to study dynamic feedbacks.

We consider the plant given by the discrete time state space equations

$$\begin{aligned} x(t+1) &= Ax(t) + Bu(t) + Ew(t) \\ y(t) &= C_y x(t) + \eta(t) \\ z(t) &= C_z x(t) + D_z u(t) \end{aligned} \qquad (1)$$

where $u$ is the control input with $u(t \in U \subseteq R^m)$and $x$ is the state with $x(t) \in R^n$.

The set **U** is a not necessarily bounded, closed, convex set which contains an open neighborhood of the origin. We assume that constraints on the state $x$ are imposed in that $x(t)$ is supposed to belong to a convex, closed set $X \subseteq R^n$ that contains the origin in its interior.

The second equation describes the measured output $y$ with $y(t) \in R^d$. The output to be controlled is $z$ with $z(t) \in R^p$. The disturbance $w$ and the measurement noise $\eta$ are two mutually independent stochastic processes with $w(t) = \mathrm{N}(0, Q_w)$ and $\eta(t) \in \mathrm{N}(0, Q_\eta)$ where $\mathrm{N}(0,Q)$ denotes the family of normally distributed random variables with zero mean and covariance matrice $Q$. Moreover, for $k \neq j$, $w(k)$ and $w(j)$ are independent as well as $\eta(k)$ and $\eta(j)$. Note that this implies that also the state $x$, the measurement $y$ and the controlled output $z$ are stochastic processes.

Thus, we consider a linear, time invariant plant, subject to stochastic disturbances with a constrained input and a constrained state variable. The measurement output $y$ is available for feedback. When the plant is subject to stochastic disturbances, the constrained input limits the ability to control the plant, as already discussed. Therefore, the following assumption is necessary.

We consider a problem of choosing $u$ such that the following cost is minimized.

$$J(x,u) := \lim_{T\to\infty} \mathrm{E}\frac{1}{T}\sum_{t=0}^{T} j(x(t),u(t)) \qquad (2)$$

subject to the state equations (1) with $x(0) = x$ where $j; R^n \times R^m \to R_+$ is a strictly convex function with $j(0, 0) = 0$. The choice of the function $j$ depends on the problem at hand. The case where only the input $u$ is constrained (i.e. $X = R^n$) has been treated. In this case, the function $j$ has been chosen as a quadratic function.The general case with constraints on the state and the input has been treated in chapter 4, where we redefined the cost $j$ so as to include an exponential penalty on state violations. Therefore, the structure of the cost (2) is general enough to capture different problems.

The control input $u$ has to be chosen such that $u(t)$ is a function of all past measurements.

Ultimately, we will wish to implement the controller by means of a digital computational device, which implies that at least 1 time unit will be required to calculate the next control action. Because of this, we assume that at time $t$ measurements $y(\tau)$, $0<\tau<t$ are used for computation of the input $u(t)$. Thus, the system (1) is controlled by means of a strictly causal dynamic feedback controller which is assumed to be representable by the state equations

$$\begin{aligned} r(t+1) &= f_{con}(r(t), y(t)) \\ u(t) &= g_{con}(r(t)) \end{aligned} \qquad (3)$$

with the initial condition $r(0)=0$ and where functions $f_{con} : R^g \times R^d \to R^g$ and $g_{con} = R^g \to R^m$ are continuous functions with $f_{con}(0,0) = 0 \quad \textit{and} \quad g_{con}(0) = 0$ and where dim$(r)$ is the (undecided) state dimension of the controller. We denote the set of all feedback controllers of the form (3) by $\sum_{con}$.

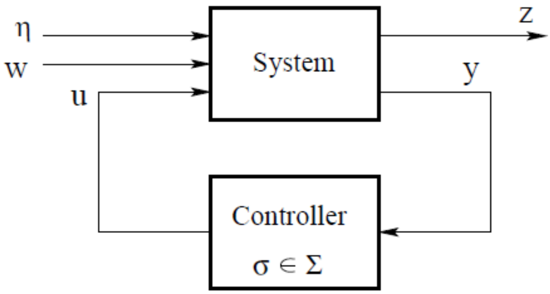

Figure 1. The system (1) is controlled by strictly causal feedback controller (3)

In general, the state of the plant is subject to constraints and we have only partial information about the state via output measurements. The standard approach is to use an optimal state observer to estimate the state of the plant. The state observer that we use for the purpose of optimal state estimation is the well known Kalman filter. A measurement feedback controller then have two separate tasks: the state estimation and the computation the optimal input that is based on the static feedback from the estimated state. Within the classical Linear Quadratic Gaussian framework, it is possible to obtain the optimal controller by this approach, according to the well known "separation principle".

In section 2 we propose a problem setup for optimal control of systems with the hard constraints on input and possible constraints on the state. When constraints on the state are present, the constraint violation cost is added to the cost function which makes the overall cost function non quadratic in general.

The problem setup does not fit in the classical LQG framework because of the input constraints and the possibly non quadratic cost function. The  separation principle" does not necessarily give an optimal controller in this case. In section 3 we study this issue and investigate in which cases the solution based on the "separation principle" gives an optimal controller and in which cases we have to find an alternative control structure.

In section 4, we design a model predictive controller that uses the optimal state estimate of the plant as an initial state for prediction. The feedback structure that is inherent to the problem (i.e. the estimated state of the plant is used for feedback) is taken into account in the prediction. The difficulty is that the output measurement is not available over the control horizon and the correction of the prediction is not possible as in the standard Kalman filtering algorithm. To overcome this difficulty, we consider the innovation of the prediction as a stochastic process. We present an algorithm for model predictive control of stochastic systems via measurement feedback.

Finally, in section 5 we present two examples in which we implement a model predictive controller developed the section 4 on the system with constrained input and the double integrator system.

***Conclusion.*** In this work we consider optimal control of linear, constrained stochastic systems via measurement feedback. We chose a controller from a set of strictly proper dynamic controllers. The controller has three main tasks: to render the closed loop system stable, to control the system so that constrains on the state are respected as much as possible and to minimize the performance measure when states are away from the constraint boundary. Since the state is not available for the measurement it is necessary to design a state estimator. The estimation has to be performed optimally, in the sense that the estimation error should have the minimum variance. This estimator is well known Kalman filter. A static feedback controller is than used to determine the input to the system, based on the estimated state.

Finally, we present an example in which we use model predictive controller developed in this chapter on the double integrator system. The simulation results show improved performance compared to the standard model predictive controller even for the relatively small number of samples.

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