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OPTIMIZATION PROBLEMS ON A SET OF POLYARRANGEMENTS

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Consider the Euclidean combinatorial set of polyarrangement $E_{\eta n}^{ks}(G,H)$ [1]. Let $G^{N_i} \subset G$ be a multiset with η_i elements ($\eta_i = |N_i|$ $\forall i \in J_s = \{1,\dots,s\}$), G^{N_i} consists of elements from G $g_1^{N_i}, g_2^{N_i}, \dots, g_{\eta_i}^{N_i}$ with numbers from the set N_i, $\eta_1 + \dots + \eta_n = \eta$; $k_1, \dots, k_s$ are natural summands, wich satisfy the condition $1 \le k_i \le \eta_i$ $\forall i \in J_s$ and in a sum give k. Let $J_n^0 = J_n \cup \{0\}$. Order without loss of generality components of the multiset G^{N_i}:

$$g_1^{N_i} \le \dots \le g_{\eta_i}^{N_i}. \tag{1}$$

Let

$$r_j, s_j \in J_{k_j}^0, \quad s_j + r_j = k_j \quad \forall j \in J_s. \tag{2}$$

Consider the problem of minimization without conditions on the set $E_{\eta n}^{ks}(G,H)$ is to find odered pair $<c^*, x^*>$, where:

$$c^* = \min_{x \in E_{\eta n}^{ks}(G,H)} \sum_{i=1}^{k} c_i x_i, \quad x^* = \arg \min_{x \in E_{\eta n}^{ks}(G,H)} \sum_{i=1}^{k} c_i x_i \tag{3}$$

and $c_j \in R^1$ $\forall j \in J_k$.

Theorem 1. Let $\beta_j = (\beta_1^j, \dots, \beta_{k_j}^j)$ is the permutation of elements of the set J_{k_j}, that is $\beta_j \in E_{k_j}^j(J_{k_j})$, for

$$c_{\beta_1^j} \ge \dots \ge c_{\beta_{s_j}^j} \ge c_{\beta_{s_j+1}^j} \ge \dots \ge c_{\beta_{k_j}^j}, \quad s_j \in J_{k_j}^0, \quad \forall j \in J_s. \tag{4}$$

Then a minimum is achieved at the point (3), which satisfies the conditions

$$x_{\beta_i^j}^* = g_i^{N_j} \ \forall i \in J_{s_j}; \quad x_{\beta_{s_j+i}^j}^* = g_{\eta_j - r + i}^{N_j} \ \forall i \in J_{r_j}, \quad \forall j \in J_s,$$

where the permutation $\beta_j \in E_k(J_k)$ and the constant $s_j \in J_k^0$ satisfy (4), elements of the multiset G satiafy the relation, constants r_j and s_j satisfy the relation (2).

Theorem 2. Let $\varphi(x)$ be a convex and finite function, given on a convex closed set $X \subset R^k$, $E^{ks}_{\eta n}(G,H) \subset X$. Then: 1) for all interior points $y \in X$

$$\min_{x \in E^{ks}_{\eta n}(G,H)} \varphi(x) \geq \varphi(y) - (p(y),y) + \sum_{j=1}^{s}\left(\sum_{i=1}^{s_j} p_{\beta^j_i}(y) g_i^{N_j} + \sum_{i=s_j+1}^{k_j} p_{\beta^j_i}(y) g_{\eta_j - k_j + i}\right), \quad (5)$$

2) the sufficien condition for the point $y \in E^{ks}_{\eta n}(G,H) \subset \operatorname{int} X$ to be the point of a minimum on the set $E^{ks}_{\eta n}(G,H)$ of the function $\varphi(x)$ is

$$p(y),y) = \sum_{j=1}^{s}\left(\sum_{i=1}^{s_j} p_{\beta^j_i}(y) g_i^{N_j} + \sum_{i=s_j+1}^{k_j} p_{\beta^j_i}(y) g_{\eta_j - k_j + i}\right),$$

where $p(y)=(p_1(y), ..., p_k(y))$ is the subgradient of the function $\varphi(x)$ at the point y, the permutation $(\beta^j_1,...,\beta^j_{k_j}) \in E^j_{k_j}(J_{k_j})$ satisfies the condition

$$p_{\beta^j_1}(y) \geq p_{\beta^j_2}(y) \geq ... \geq p_{\beta^j_{s_j}}(y) \geq 0 > p_{\beta^j_{s_j+1}}(y) \geq ... \geq p_{\beta^j_{k_j}}(y),$$

and elements of the multiset G are ordered according to (1).

The above account provides a universal approach to the estimation of global extrema on Euclidean combinatorial sets of known convex functions. The examples of a meaningful formulation and the mathematical model of a problem, the set of admissible solutions of which is the Euclidean polyarrangement set are considered in [2]. Accounts of the special case of a polyarrangement set (in the cases of sets of permutations, polypermutations, arrangements) with meaningful formulations and corresponding models can be found, for instance, in [1].

The results given in this paper can be used when employing various combinatorial optimization methods. The sufficient conditions obtained provide the possibility of proving global optimization, and the estmates for z^0 proved here (z^0 is the right-hand side of inequality (5) or a similar inequality) enable one to find bounds for the absolute and relative errors $\Delta = \tilde{z} - z^*$ and $\delta = \Delta / |z^*|$ of an approximate solution $\tilde{z}$ in the various local optimization algorithms on the set $E^{ks}_{\eta n}(G,H)$ and its specific realizations (where z^* denotes the minimum value of the objective function on $E^{ks}_{\eta n}(G,H)$). In fact, by virtue of the inequality $z^0 \leq z^* \leq \tilde{z}$, we have $\Delta = \tilde{z} - z^* \leq \tilde{z} - z^0$, and for $z^0>0$, we have $\delta = \Delta / z^* \leq \tilde{z} / z^0 - 1$.

References

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